

Answer any TWO of the following:

- 19. Diagonalize the matrix $A = \begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & 1 \end{pmatrix}$. (20)
- 20. a) Show that the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ satisfies the equation $A^2 = -I$. Hence calculate the 16^{th} power of $B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

b) Find the maxima and minima of the function $y = 2x^3 - 3x^2 - 36x + 10$. (12+8)

- 21. Given a vector-valued function $\vec{F} = (x^3 yz)\vec{i} 2x^2y\vec{j} + 2\vec{k}$, verify Gauss Divergence theorem for \vec{F} over the cube bounded by x = 0, y = 0, z = 0, x = a, y = a, z = a. (20)
- 22. a) Solve $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \log x$.
 - b) Find the general solution of $x^2p + y^2q = (x + y)z$. (10+10)

 $(2 \times 20 = 40)$